NOVEL PARALLEL QUANTUM-INSPIRED EVOLUTIONARY ALGORITHMS FOR OPTIMIZATION

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1. Introduction

Algorithms are designed for finding the best possible solution(s) to a given problem at hand. The problems could be related to any discipline of human culture, nature, modeling and system. Evolutionary Algorithms (EAs) are inspired by natural evolution and have been used for several decades for solving difficult optimization problems which are known to give near optimal solutions. A variety of EAs have been designed with differences in implementation according to the requirements of the particular problem being attempted. There are problems in various classical EAs such as large storage requirements for calculations, slow convergence, local optimization, less efficiency, inability to find the best possible solutions, etc. To overcome these shortcomings in EAs, researchers have attempted to introduce ideas from Quantum Computing (QC) into them.

In QC, a system may be in a superposition of all possible ground states. These superposition states are proposed to be processed in parallel by a quantum computer. This is an interesting feature exploited by computer scientists. QC can provide exponential speedup over conventional computers. Of course, this speedup is available only under special circumstances through specially designed quantum algorithms. Many EAs have been redesigned to possess some quantum features along with their classical capabilities in order to get the advantages of QC. Thus, there are many quantum inspired algorithms which have been developed by enhancing an existing computing paradigm by incorporating quantum features in various ways.

The superposed state is a superposition of ground states with well-defined probability of obtaining each ground state when observed. A Quantum Bit or qubit is the smallest unit of quantum information analogous to the bit in classical computing. Thus, a single memory unit with required number of qubits can represent all the possible solutions with a finite probability of generating each one of them when measurement or observation is made [1].

It means there is a need of some reasonable operations to modify the probabilities as desired in the algorithm being implemented so that the desired solution is obtained on observation. Typically operators called Quantum Gates are utilized for this purpose. Quantum Gates are operators that allow transformation of qubit states into other states with or without the help of some randomized component.

2. Motivation

There has been tremendous spurt in the research activity such as community detection in complex social networks by optimizing modularity[92**Error! Reference source not found.**], real coded synthetic and real world optimization problems [93], function optimization for high dependence problems [94], Sonar image detection[9594], real-time tasks scheduling in multiprocessor environment[96], integrated process planning and scheduling[9797],parameter analysis in convergence quality[98], link prediction in complex network strictures [99] in this area which provides extensive motivation for a deeper study that would encompass both algorithmic advances in terms of including more idea pertaining to quantum computation and more detailed experimental work for obtaining best performance over a larger variety of problems.

As problems become larger and complex with time and require the high computation cost, it is hard to deal with the serial optimization algorithms, some parallel and distributed QIEAs are also needed to employ for solving such complex problems that process big data requiring high computation cost. These endeavors appear to be particularly attractive due to the availability of high performance CUDA graphic cards with ever increasing number of cores at lower prices.

3. The Basic Quantum-Inspired Evolutionary Algorithm

Han and Kim [2] proposed an algorithm that utilized Quantum Bit or qubit representation and other features in the classical EA and called it the Quantum-Inspired Evolutionary Algorithm (QIEA). Since this algorithm is prototypical of several such efforts, this version developed by Han and Kim is called the Basic QIEA. The algorithm is succinctly described in the form of pseudo-code as follows.

Algorithm: Pseudocode algorithm of the QIEA [2]
Require: An initial population Q(t), t = 0
1: Make P(t) by observing the states of Q(t)
2: Evaluate P(t)
3: Store all solutions in P(t), into B(t) and the best solution b in B(t)
4: while (not termination condition) do
5: $t \leftarrow t + 1$
6: Make P(t) by observing the states of Q(t − 1)
7: Evaluate P(t)
8: Update Q(t) using Q-gates
9: Store all solutions in P(t), into B(t − 1) and the best solution b in B(t)
10: if (migration condition) then
11: Migrate b or b_j^t to B(t) globally or locally, respectively
12: end if
13: end while

Notations used in the standard QIEA are-

Q(t):- Qubit population in t^{th} iteration.

P(t):- population of binary solutions in t^{th} iteration.

B(t):- population of best solutions found till t^{th} iteration.

q_j^t:- j^{th} individual in Q (t) .

p_j^t:- j^{th} individual in P (t) .

b_j^t:- j^{th} individual in B (t) .

b:- best solution observed so far.

n:- Number of items to be considered in problem.

t:- the current iteration.

Following are the various versions of QIEAs reported in the literature-

(i) Motivated by the inherent parallelism in qubit representation, Han and Kim proposed QGA for knapsack problem, without crossover and mutation to avoid performance reduction due to high probabilities [3].

(ii) In order to meet the higher population demand, Han et al. [4] proposed parallel QGA. Parallel QGA based on repair methods was used to obtain optimized solution in shorter computing time over QGA.

(iii) Since the update strategy of Q-gates depends on prior knowledge of the criterion of the optimal solution, QGA for Knapsack and parallel QGA were not suitable for solving continuous optimization problems, specifically multi-peak continuous problems. To overcome such problems, an extended coarse-grained hierarchical ring model in CGPQGA (Coarse-Grained Parallel QGA) was proposed by Zhang et al. [5].This model was used for the design of IIR Filters in which phase comparison method was used as the update strategy of Q-gates anda search grid was made based on phase comparison method. The main advantage of CGPQGA was to eradicate the disadvantages in existing parallel algorithms such as communication overheads and delayed migration of best individual.

(iv) A novel QIEA was presented by Patvardhan et al. [6]for the Quadratic Knapsack Problem. It is shown to perform competitively on benchmark QKP instances and is indicative of the effectiveness of the QIEA for the problem.

(v) A hybrid QIEA, dubbed as QIEA-MKP, was presented by Patvardhan et al. [7] for multiple knapsack problem.

(vi) An improved version of QIEA for quickly solving extremely large spanner problem instances was presented in [8].

(vii) Ni and Wang [9]employed niche elimination mechanism to maintain diversity and to avoid premature convergence while dealing with multi-peak problems.

(viii) Xu et al. [10]replaced the approximation operator by Quantum rotation gate.

(ix) Recently QGA was employed for multi-objective optimization problems such as dynamic alliance and slope detection with Wireless Sensor Networks (Wang et al. [11], Liu [12-13]).

4. Nature-inspired QIEAs

Few other popular Nature-inspired EAs are Particle Swarm Optimization, Ant Colony Optimization, Artificial Fish-Swarm algorithm and Immune Systems. Table 1 depicts various real-world applications where different nature-inspired QIEAs are employed and better performance than classical algorithms is claimed. Though all this quoted work is quantum-inspired, the quantum phenomena used are very few, typically limited to Quantum state representation of the search population and Rotation Gates only. Some approaches that use other Quantum features are described in section 4 [14-16].

Sr. No.	Application	Nature–inspired QIEAs	Quantum-features used	References
1	Bioinformatics	PSO	Quantum state representation of a particle and Rotation Gate	[18-22]
2	Design	PSO	--Do--	[23-34]
		ACO	--Do--	[35-36]
		IS	--Do--	[37]
3	Benchmark problems	PSO	--Do--	[38-39]
		ACO	--Do--	[40]
		IS	--Do--	[41-43];
4	Benchmark functions	PSO	--Do--	[44-55]
		AFSA	--Do--	[56-57]
		IS	--Do--	[58-60]
5	Control	PSO	--Do--	[61-68]
		IS	--Do--	[69]
8	Signal processing	PSO	--Do--	[70-73]
		IS	--Do--	[74]
9	Scheduling	PSO	--Do--	[75-77]
		IS	--Do--	[78-79]
10	Robotics	IS	--Do--	[80-81]
11	Image and video	PSO	--Do--	[82-85]
		IS	--Do--	[86-87]
12	Graphics and visualization	PSO	--Do--	[88]

Table 1: Some Applications of Nature-inspired QIEAs with limited Quantum Phenomena

4.1 Quantum – inspired particle swarm optimization

Since Particle Swarm Optimization (PSO) does not require the problem to be differentiable, it can be applied on optimization problems that are partially irregular, noisy, change overtime etc. Though PSO works well for linear

problems, it is not suitable for many complicated problems. Further, global convergence is not guaranteed and it may get trapped in local minimum. Hence, it is not suitable for discrete optimization. Ideas from QC are introduced into PSO to depict the uncertainties in thinking thereby forming Quantum-inspired PSO (QPSO).

In section 1 of Table 1, the utilization of QPSO in the field of Bioinformatics is shown. A new method is proposed, called Diversity-Maintained QPSO (DMQPO), in which is a diversity control strategy is incorporated into QPSO to enhance the global search ability [18]. In another work, a QPSO is employed to obtain optimization of codon usage [19]. An improved quantum evolutionary algorithm combined with chaos is proposed to solve DNA encoding [20]. A Multiple-layer QPSO is proposed for protein structure prediction in [21] and a Hidden Markov Model is combined with QPSO for multiple sequence alignment in [22].

In Section 2 of Table 2, various versions of QPSO are employed to solve design problems such as Gaussian QPSO for the electromagnetic design [23].A QPSO with chaotic sequences as mutation operator to solve a continuous optimization problem of mechanical engineering design is presented in [24]. A QPSO with Gaussian probability distribution for solving continuous optimization problems of engineering design is presented in [25]. A PSO-QI(Particle Swarm Optimization with Quantum Infusion) is proposed to solve design problems and is called so because QPSO has been used(infused) to update and guide the *gbest* particle obtained from the traditional PSO [26]. A Chaos Quantum-behaved Particle Swarm Optimization algorithm (CQPSO) is proposed and a Chaos Quantum-behaved Particle Swarm Constrained Optimization algorithm program with hybrid Discrete variables (DCQPSO 1.0) is developed, to deal with value adopting problems of hybrid discrete variables in optimization design [27]. Another QPSO is implemented and is used in FIR digital filters in [28]. An improved QPSO (called Diversity-Guided QPSO) is proposed and used for 2-D IIR filter design. The diversity in DGQPSO is measured by average Euclidean distance from the particle's current position to its centroid swarms [29-30]. A new QPSO using the exponential probability distribution called E-QPSO is used for the mutation operator [31]. A QPSO algorithm is employed for optimization of the Phased array feeds that are modeled as integer programming problem in [32]. A model for multi-objective design optimization of laminated composite components using a novel multi-objective optimization algorithm is developed on the basis of the Quantum behaved Particle Swarm Optimization (QPSO) [33]. A QPSO procedure is proposed for the design of virtual

manufacturing cells within which machines and jobs are assigned to the cells with a view to maximize productive output, whilst simultaneously minimizing the inter-cell movements due to the limited availability of machines [34].

Section3 of Table-1shows that a Multi-Objective Quantum-inspired Particle Swarm Optimization (MOQPSO) consisting Gaussian distribution probability operator has been proposed and results have been reported to show that MOQPSO is capable of finding the non-dominated solutions i.e. a set of solutions in which no solution is better than any of the others in respect of all the objectives [38]. A version of Multi-Swarm Quantum Optimization algorithm (mQSO) is presented by including heuristic knowledge to improve its performance [39]. Section4 of Table 1 shows the utilization of different QPSOs on different benchmark functions. A QPSO is implemented on Map Reduce model and Map Reduce Quantum-behaved Particle Swarm Optimization (MRQPSO) is proposed which achieves parallel and distributed QPSO in [44]. Diversity-guided quantum-behaved particle swarm optimization algorithm based on clustering coefficients and characteristic distance is proposed and numerical optimization simulation results demonstrate better optimization performance in [47]. An improved QPSO algorithm, dubbed IQPSO, by combining QPSO and an opposition-based learning concept is proposed in [48]. A Quantum-behaved Particle Swarm Optimization with Mutation operator (MQPSO) to solve seismic wave impedance inversion problem is proposed in [49]. The QPSO is used to solve various problems such as constraint-handling multi-objective optimization [50-51], function optimization [52-53] etc.

Various Parallel Quantum Evolutionary Algorithms based on quantum computation and chaotic searching technique have been proposed and used to obtain the multi-modal optimal solutions and global optimization [58-60].

Quantum-behaved particle swarm optimization is employed for various control application such as Online system identification, Output feedback UPFC controller design, Thruster fault-tolerant for UUVs, Tuning PID control parameters on hydraulic servo control system, Tuning of damping controller for UPFC, Maximization of

expected target damage value, aero engine maximum thrust optimization, Aircraft evasive maneuver trajectory optimization and better performance is indicated using QPSO [61-68].

The utilization of quantum-behaved-nature-inspired evolutionary algorithms is shown for signal processing in section 8 of Table 2 such as QPSO employment for MC-CDMA multiuser detection [70-73]. Another application of quantum-behaved nature-inspired evolutionary algorithm is in scheduling is shown in section 9 of Table 2. An improved Quantum-inspired particle swarm optimization (QPSO) algorithm is employed for rail guided vehicles system (RGV)in[75]. A Quantum behaved Iterated Greedy algorithm (QIG) with a new rotation gate based on Particle Swarm Optimization (PSO) is employed for permutation flow-shops to minimize the total flow-time [76] and a QPSO was proposed for the no-wait flow shop scheduling problem with makespan criterion [76].

The design of T-S fuzzy control for magnetic levitation systems is proposed by using PSO and QEA to search the optimal feedback gains in [80].

Sections 11&12 of Table 1 show the work regarding employment of quantum-inspired PSO and Immune System for images and videos, and, graphics and visualization applications. These applications include Medical image registration [82], Multilevel thresholding for image segmentation [83], image alignment [84], Multilevel maximum entropy threshold[85], multi-scale Bandelet-based image representation [86], Image segmentation [87] and detection of unstable periodic orbits of nonlinear mappings [88].

4.2 Quantum-inspired Ant Colony Optimization

A metaheuristic algorithm inspired from the behavior of real ant colonies, Ant Colony Optimization (ACO), has been used effectively to solve discrete optimization problems. However, it has certain disadvantages also such as premature convergence, low searching ability and slow convergence speed. To resolve these, the Quantum-inspired ACO (QACO) is formulated.

A novel self-adaptive Ant Colony Optimization algorithm with inclusion of quantum phenomena (TQACO) is proposed to solve the Traveling salesman problem [40].QACOs could also be employed to solve various design

problems. A design method for induction motor has been suggested based on continuous quantum ant colony optimization [35-36].

4.3 Quantum-Inspired Artificial Fish-Swarm algorithm

Artificial Fish Swarm Algorithm (AFSA) is another Swarm Intelligence algorithm inspired by Fish Schools. In AFSA, the pattern of the Fish finding the nutritious areas irrespective o fits initial position is constructed. This algorithm too has disadvantages such as decrease in convergence speed at the later stage of the algorithm, getting trapped in local optimum etc. These motivate inclusion of Quantum phenomena in to AFSA.

A novel Quantum Artificial Fish Swarm Algorithm (QAFSA), based on quantum computing, is proposed to increase the diversity of species, fast convergence and better global search [56-57].

4.4 Quantum-Inspired Immune systems

Artificial Immune System (AIS) is a computational system, inspired by the principles and processes of the vertebrate immune system. In order to speed up the convergence, the Quantum-inspired Immune System is formulated. An IS method for motor structure optimization has been suggested based on continuous quantum immune clonal optimization [37].

A Quantum Immune Clonal Co-evolutionary Algorithm (QICCA) is proposed incorporating quantum evolutionary computing and the strategy of co-evolution in the AIS to solve dynamic multi-objective optimization problems [41].

Based on quantum computing and immune system, a novel quantum immune algorithm for finding Pareto-optimal solutions to multi-objective optimization problems with a few distinctive features such as qubit representation of population, mutation based on chaos-based rotation gate and Q-gates and a new truncation algorithm with similar individuals (TASI) is utilized to preserve the diversity of the population [43].

A novel train traffic control method based on time petri-nets and immune quantum optimization algorithm is proposed for controlling the train traffic [69].Quantum-inspired immune clonal optimization for multiuser detection in direct-sequence code-division multiple-access (DS-CDMA) is proposed in[74].A quantum clone immunity algorithm was introduced to solve the problem of scheduling heterogeneous multiprocessor system [78].A quantum-inspired immune algorithm (QIA) is proposed for Hybrid flow shop problems (HFSP) to minimize makespan [79].

Quantum-Inspired Immune Systems (QISs) are also employed in the field of Robotics as shown in section 10 of Table 1.A multi-objective optimization framework, with an immune-inspired algorithm dubbed Suppression-Controlled Multi-objective Immune Algorithm (SCMIA) and a simulation model for solving a real-life multi-objective optimization problem is employed for solving multi-objective simulation-based optimization problem with Material Handling System (MHS) [81].

5. Utilization of other quantum phenomena in QIEA

This literature review shows that most of the papers have used only limited quantum capabilities, being limited to qubit representation and rotation gates only. Very few researchers have extended the inclusion of quantumness. Dai and Li modified the Crossover operator of conventional GA using quantum interference [14].Yang et al. adopted “one-to-one” emigration strategy and quantum cross over for the Information Exchange among Multi-universe parallel structures, separately [15] and with Independent Component Analysis [16], to obtain the global optimal solution of Nonlinear Blind Source Separation and Blind Source Separation algorithm respectively.

To meet certain problems with normal qubit representation such as nonsuitability of binary Coding for numerical optimization, randomness in probabilistic operations, uncertainty in determining the magnitude rotational angles of quantum gatesetc., encoding of qubit is done through the Bloch coordinates [17].

In this section, the work containing quantum phenomena other than qubit representation and rotation gate is shown in Table 2.

Quantum Phenomena other than qubit representation and rotation gates	**Used in QIEA**	**Reference**
Quantum interference	To modify the Crossover Operator of GA	[14]
Quantum cross over for the Information Exchange among Multi-universe parallel structures	To obtain the global optimal solution of Nonlinear Blind Source Separation and Blind Source Separation algorithm	[15-16]
Qubit is encoded using Bloch coordinates.	To decrease the blind characteristics of qubit measurement.	[17]

Table 2:- QIEAs with Quantum Phenomena other than qubit representation and rotation gates.

6. GPGPU implementation of Evolutionary Algorithms (EA)

Evolutionary algorithms have been implemented on GPGPU but with limited success as reported in the literature. One of the main roadblocks in this endeavor is that EAs typically need to be implemented with large populations to ensure exploration of the complete search space. The convergence speed and communication overheads become difficult issues since the populations have to be collected together and then communicated back to the single GPU creating enormous overheads. The QIEAs implementation on GPGPU would be investigated to mitigate these issues.

EAs are inherently parallel due to their being population based. A coarse-grained parallel implementation can be easily implemented using an MPI architecture wherein an island model of parallel GA can be implemented. In this model, each CPU core or node runs the GA for a single population with periodic transfer of the best solution found so far to all the nodes. This may enable non-linear speed up because all populations benefit even if one good solution is formed by a node. Such parallel implementations are suited for large clusters of machines such as LINUX Cluster with MPI environment. These are expensive to create although good speedups may be obtained using this strategy on some problems.

Recently, the availability of massively parallel architectures of modern Graphics Processing Units(GPUs) has been on the rise. These provide thousands of cores on a single GPU and with the limitation being that each core has only very limited local memory.

GPUs were first designed in 70s to act as a co-processor for the rendering of images on Graphics/Video cards. There are increasingly being utilized for "Generic" General Purpose Computing or GP-GPUs. CUDA (NVIDIA's proprietary Complete Unified Device Architecture) is one of the most common GP-GPUs [89]. EAs have recently been attempted on GP-GPUs [90]. Hofmann et al. [91] highlights the different models used for transferring GAs as GP-GPU computations, namely the island model, the cellular model and the master worker model. Each of these models has its own advantages and shortcomings.

7. Proposed Work

The above literature survey highlights the large body of work that has been reported in this promising area of using ideas from QC and EAs together to develop powerful search and optimization algorithms. However, it also throws up some gaps in these research endeavors and an attempt shall be made in this thesis to fill some of these gaps. Specifically the following shall be attempted:

(i) The existing literature highlights the facts that the most of existing QIEAs have utilized very limited features like rotation gates and qubit representation. An attempt shall be made to design truly quantum-inspired algorithm by considering more robust gates and operators that exploit the quantum representation more effectively. Impact of utilizing hierarchy of populations with lower level populations providing the guidance towards promising regions could be explored. These ideas are possible to be implemented with QIEAs because of very

(ii) The survey of QIEAs presented above also highlights the fact that most of the work proposed in the literature restricts the quantum inspiration to qubit representation of the solution string. Quantum Theory offers many other distinctive features like entanglement, teleportation, non-local interaction among others. Attempts would be made to incorporate some of these quantum ideas in QIEA to make them better for search and optimization.

(iii) A host of attempts have been reported in the literature that present "Quantum-inspired" versions of the various Nature-inspired algorithms like Particle-Swarm Optimization and/or Artificial Bee Colony algorithm etc. However, these also have very limited "Quantum-inspired" ingredients and mostly stick to quantum representation alone. Attempt shall be made in this work to incorporate ideas developed in (i) and (ii) above to enhance the search power of the "Quantum"-cum-"nature-inspired" algorithm under consideration.

(iv) Many models have been reported for the coarse-grained parallelization of EAs each having its own advantages and disadvantages. Attempts shall be made to investigate parallel models for QIEAs which have their own specific characteristics described above and their parallelization would be studied separately.

(v) Implementing a GA using fine grained parallelism on a GP-GPU is a non-trivial task because large speedups are hard to get. The major issue with EAs typically need to process large populations specially when solving problems with large solution spaces. Further, several local search heuristics need to be incorporated for getting the desired speedups in convergence to the optimal solution. These make it difficult to obtain parallelization of the desired order although some successes have been reported in the literature. In contrast QIEAs can be implemented with very small populations-even with a population size of 1. The basic QIEA of Han and Kim envisages local migration as an in frequently utilized mode. A suitable implementation is, therefore, visualized on GP-GPUs utilizing the very large number of cores that are available and keeping the communication overheads to manageable levels at the same. QIEAs implementation has not yet been attempted on GP-GPUs and an attempt shall be made to fill this gap in this work.

(vi) All the implementations described above shall be explored for their computational efficiency on standard problems that serve as benchmarks in the areas of Image processing, Combinatorial and Function optimization etc.

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